

B.Sc. Part-II, Paper IV.

(Group B) [VECTOR CALCULUS]

Scalar Triple Product

•  $\vec{a} \cdot (\vec{b} \times \vec{c}) = [\vec{a} \vec{b} \vec{c}]$  is a scalar quantity.

•  $\vec{a} \cdot (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \cdot \vec{c} = \vec{b} \cdot (\vec{c} \times \vec{a})$   
 $= \vec{c} \cdot (\vec{a} \times \vec{b}) = (\vec{b} \times \vec{c}) \cdot \vec{a} = (\vec{c} \times \vec{a}) \cdot \vec{b}$

That means dot and cross can be interchanged.

Also, the position of  $\vec{a}, \vec{b}, \vec{c}$  can be changed

as  $\vec{a}$  to  $\vec{b}$ ,  $\vec{b}$  to  $\vec{c}$ ,  $\vec{c}$  to  $\vec{a}$

•  $[\vec{a} \vec{b} \vec{c}] = -[\vec{b} \vec{a} \vec{c}]$

• If two vectors are identical, then the scalar triple product = 0

i.e.  $[\vec{a} \vec{a} \vec{b}] = 0 = [\vec{a} \vec{b} \vec{a}] = [\vec{a} \vec{b} \vec{b}]$

•  $[\vec{i} \vec{j} \vec{k}] = 1$

•  $[\vec{a} \vec{a} \vec{a}] = 0$

• If the vectors  $\vec{a}, \vec{b}, \vec{c}$  are coplanar, then

$[\vec{a} \vec{b} \vec{c}] = 0$

• If  $\vec{a} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$ ,  $\vec{b} = b_1\vec{i} + b_2\vec{j} + b_3\vec{k}$

$\vec{c} = c_1\vec{i} + c_2\vec{j} + c_3\vec{k}$ , then

$$[\vec{a} \vec{b} \vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$



Ex 1. Find  $(2\vec{i} + 3\vec{j} - 5\vec{k}) \times (3\vec{i} - 2\vec{j} + 6\vec{k}) \cdot (5\vec{i} + \vec{j} + \vec{k})$

Soln. The given expression =  $\begin{vmatrix} 2 & 3 & -5 \\ 3 & -2 & 6 \\ 5 & 1 & 1 \end{vmatrix}$   
 $= \begin{vmatrix} 5 & 1 & 1 \\ 3 & -2 & 6 \\ 5 & 1 & 1 \end{vmatrix}$  By  $R_1 \rightarrow R_1 + R_2$   
 $= 0$

Ex 2. Find  $(\vec{a} \times \vec{b}) \cdot \vec{c}$  where  $\vec{a} = 2\vec{i} - 3\vec{j} - \vec{k}$ ,  $\vec{b} = 2\vec{i} + \vec{j} - \vec{k}$ ,  
 $\vec{c} = \vec{i} - \vec{j} + 2\vec{k}$ .

Soln.  $\vec{a} \times \vec{b} = (2\vec{i} - 3\vec{j} - \vec{k}) \times (2\vec{i} + \vec{j} - \vec{k})$   
 $= -6\vec{j} \times \vec{i} - 2\vec{k} \times \vec{i} + 2\vec{i} \times \vec{j} - \vec{k} \times \vec{j} - 2\vec{i} \times \vec{k} + 3\vec{j} \times \vec{k}$   
 $= 6\vec{k} - 2\vec{j} + 2\vec{k} + \vec{i} + 2\vec{j} + 3\vec{i}$   
 $= 4\vec{i} + 8\vec{k}$

$\therefore (\vec{a} \times \vec{b}) \cdot \vec{c} = (4\vec{i} + 8\vec{k}) \cdot (\vec{i} - \vec{j} + 2\vec{k})$   
 $= 4 \times 1 - 0 + 8 \times 2 = 20$

Volume of a parallelepiped whose edges are  $\vec{a}, \vec{b}, \vec{c}$   
 $= \vec{a} \cdot (\vec{b} \times \vec{c}) = [\vec{a} \ \vec{b} \ \vec{c}]$

Q. Find the volume of a parallelepiped whose edges are  $\vec{i} + 2\vec{j} + 3\vec{k}$ ,  $3\vec{i} + 7\vec{j} - 4\vec{k}$  and  $\vec{i} - 5\vec{j} + 3\vec{k}$ .

Soln. The volume of a parallelepiped  
 $= (\vec{i} + 2\vec{j} + 3\vec{k}) \cdot (3\vec{i} + 7\vec{j} - 4\vec{k}) \times (\vec{i} - 5\vec{j} + 3\vec{k})$   
 $= \begin{vmatrix} 1 & 2 & 3 \\ 3 & 7 & -4 \\ 1 & -5 & 3 \end{vmatrix}$  By  $R_1 \rightarrow R_1 - R_3$   
 $= \begin{vmatrix} 0 & 7 & 0 \\ 3 & 7 & -4 \\ 1 & -5 & 3 \end{vmatrix} = -7 \begin{vmatrix} 3 & -4 \\ 1 & 3 \end{vmatrix} = -7(9+4) = -91$

Since, volume cannot be negative, so the required volume = 91 cubic unit.